

UT-725  
October, 1995

## On Effective Superpotentials in Supersymmetric Gauge Theories

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### ABSTRACT

Effective superpotentials are considered which include glueball chiral superfields among their arguments in supersymmetric gauge theories. It is seen that the accommodation of glueball superfields is suitable for their complete determination.

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Symmetries provide exact information on effective actions for field theories. In particular, supersymmetry is powerful enough to sometimes determine effective superpotentials<sup>[1]</sup> even uniquely.<sup>[2,...,9]</sup>

In gauge theories, certain internal symmetries turn out to be anomalous. There have been two ways for dealing with anomalous symmetries to determine effective superpotentials in supersymmetric gauge theories. One way given by Taylor, Veneziano, and Yankielowicz<sup>[3]</sup> is to introduce glueball chiral superfields<sup>[2]</sup> to realize anomalous transformation laws for effective actions. The other way given by Intriligator, Leigh, and Seiberg<sup>[8]</sup> is to view anomalous symmetries as explicitly broken and utilize them to derive selection rules<sup>[6]</sup> for effective superpotentials.

In this paper, we take advantage of the former way in determination of effective superpotentials. Two simple examples are presented for concreteness.

Let us first consider supersymmetric  $SU(N_c)$  gauge theory with  $N_f = N_c + 1$  flavors of quark chiral superfields  $Q_i$  and  $\overline{Q}^i$  in the fundamental representations  $N_c$  and  $\overline{N}_c$ , respectively, where  $i$  denotes the flavor index:  $i = 1, \dots, N_f$ . We put a mass term for the  $N_f$ -th flavor

$$W_{tree} = m Q_{N_f} \overline{Q}^{N_f} \quad (1)$$

as a tree-level superpotential.

We take a glueball superfield<sup>[2]</sup>

$$S \sim W_\alpha W^\alpha, \quad (2)$$

where  $W_\alpha$  is the field-strength chiral superfield for the  $SU(N_c)$  gauge multiplet,

in addition to the meson superfield  $M_i^j$  and the baryon superfields  $B^i, \overline{B}_j$  as variables<sup>[5,7]</sup> to describe an effective superpotential  $W_{eff}$ .

Now we introduce a term

$$W_{dyn} = -S \ln[S^{-1} \Lambda^{-2N_c+1} (\det M - B^i M_i^j \overline{B}_j)] - S, \quad (3)$$

which satisfies all the symmetry constraints including an anomalous one in the theory with the mass  $m$  regarded as a spurion superfield.<sup>[6]</sup> Here  $\Lambda$  denotes a dynamical scale of the gauge interaction.

Then we may write the effective superpotential as follows:

$$W_{eff} = W_{dyn} + S f \left( \frac{m M_{N_f}^{N_f}}{S}, \frac{B^i M_i^j \overline{B}_j}{\det M} \right), \quad (4)$$

where  $f$  is a holomorphic function to be determined. This is because  $W_{dyn}$  saturates the anomaly and hence  $W_{eff} - W_{dyn}$  satisfies all the ‘classical symmetries’ in the theory.

The term  $W_{dyn}$  exclusively leads to an effective superpotential

$$W'_{dyn} = \Lambda^{-2N_c+1} (B^i M_i^j \overline{B}_j - \det M), \quad (5)$$

when the superfield  $S$  is integrated out. Since this superpotential is appropriate<sup>[7]</sup> for the massless limit with  $N_f = N_c + 1$ , we impose a condition that  $f = 0$  for  $m = 0$ . Then we get

$$f(x, y) = x, \quad (6)$$

provided the perturbation theory works in the asymptotic regime  $\Lambda \rightarrow 0$  (and

$m \rightarrow 0$ ). Thus we conclude

$$W_{eff} = W_{dyn} + m M_{N_f}^{N_f}. \quad (7)$$

Note that the expression (5), in contrast to (3), seems inappropriate in considering the asymptotic regime due to its singularity for  $\Lambda \rightarrow 0$ .

Let us turn to the next example. We consider supersymmetric  $SU(2)_1 \times SU(2)_2$  gauge theory with a chiral superfield  $\Phi$  in a representation  $(\mathbf{2}, \mathbf{2})$ , whose dynamical scales are given by  $\Lambda_1$  and  $\Lambda_2$ . We take glueball superfields  $S_1$  and  $S_2$  corresponding to the gauge groups  $SU(2)_1$  and  $SU(2)_2$  in addition to a gauge-singlet chiral superfield

$$X \sim \Phi\Phi \quad (8)$$

as variables<sup>[8]</sup> to describe an effective superpotential  $W_{eff}$ .

We introduce an anomaly-saturating term

$$W_{dyn} = S_1 \ln \frac{\Lambda_1^5}{S_1 X} + S_2 \ln \frac{\Lambda_2^5}{S_2 X} - S_1 - S_2 + S_1 F\left(\frac{S_2}{S_1}\right), \quad (9)$$

where  $F$  denotes a holomorphic function to be determined.

We can add a tree-level mass term for  $\Phi$  with mass  $m$  to obtain an effective superpotential

$$W_{eff} = W_{dyn} + S_1 f\left(\frac{mX}{S_1}, \frac{S_2}{S_1}\right), \quad (10)$$

where  $f$  is a holomorphic function which satisfies a condition that  $f = 0$  for  $m = 0$ .

By means of the asymptotic limit  $\Lambda_1, \Lambda_2 \rightarrow 0$  (and  $m \rightarrow 0$ ), we get

$$W_{eff} = W_{dyn} + mX. \quad (11)$$

Integrating out the superfield  $X$ , we obtain

$$W'_{eff} = S_1 \ln \frac{m\Lambda_1^5}{S_1(S_1 + S_2)} + S_2 \ln \frac{m\Lambda_2^5}{S_2(S_1 + S_2)} + S_1 F. \quad (12)$$

On the other hand, the massive theory is expected to yield pure supersymmetric  $SU(2)_1 \times SU(2)_2$  gauge theory as its low-energy limit, whose effective superpotential is given by<sup>[2]</sup>

$$W_{pure} = S_1 \ln \frac{\Lambda_1'^6}{S_1^2} + S_2 \ln \frac{\Lambda_2'^6}{S_2^2}, \quad (13)$$

where  $\Lambda_1'$  and  $\Lambda_2'$  are dynamical scales for the pure gauge theory. Comparing this expression with (12), we see

$$S_1 F = S_1 \ln \frac{S_1 + S_2}{S_1} + S_2 \ln \frac{S_1 + S_2}{S_2}. \quad (14)$$

More generally, the above approach confirms the linearity<sup>[8,9]</sup> of an effective superpotential

$$W_{eff} = W_{dyn} + W_{tree}, \quad (15)$$

where  $W_{dyn}$  is an anomaly-saturating term independent of the couplings in a tree-level term  $W_{tree}$ .

In conclusion, we have seen that it is suitable for determination of effective superpotentials in supersymmetric gauge theories to introduce glueball superfields among their arguments from the beginning.

## ACKNOWLEDGEMENTS

We would like to thank H. Murayama, and T. Yanagida for valuable discussions.

## REFERENCES

1. M.A. Shifman and A.I. Vainshtein, *Nucl. Phys.* **B277** (1986) 456; **B359** (1991) 571;  
M. Dine and Y. Shirman, *Phys. Rev.* **D50** (1994) 5389;  
S.P. de Alwis, hep-th/9508053.
2. G. Veneziano and S. Yankielowicz, *Phys. Lett.* **B113** (1982) 231;  
G.M. Shore, *Nucl. Phys.* **B222** (1983) 446;  
C.P. Burgess, J.-P. Derendinger, F. Quevedo, and M. Quirós, hep-th/9505171.■
3. T.R. Taylor, G. Veneziano, and S. Yankielowicz, *Nucl. Phys.* **B218** (1983) 493.
4. A.C. Davis, M. Dine, and N. Seiberg, *Phys. Lett.* **B125** (1983) 487;  
I. Affleck, M. Dine, and N. Seiberg, *Nucl. Phys.* **B241** (1984) 493; **B256** (1985) 557.
5. J.-M. Gérard and J. Weyers, *Phys. Lett.* **B146** (1984) 411;  
D. Amati, K. Konishi, Y. Meurice, G.C. Rossi, and G. Veneziano, *Phys. Rep.* **162**■ (1988) 169.
6. N. Seiberg, *Phys. Lett.* **B318** (1993) 469;  
K. Intriligator and N. Seiberg, hep-th/9509066.
7. N. Seiberg, *Phys. Rev.* **D49** (1994) 6857.
8. K. Intriligator, R.G. Leigh, and N. Seiberg, *Phys. Rev.* **D50** (1994) 1092.
9. V. Kaplunovsky and J. Louis, *Nucl. Phys.* **B422** (1994) 57;  
K. Intriligator, *Phys. Lett.* **B336** (1994) 409;

K. Intriligator and N. Seiberg, hep-th/9506084.